Final state interactions in two-particle interferometry

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We reconsider the influence of two-particle final state interactions (FSI) on two-particle Bose-Einstein interferometry. We concentrate in particular on the problem of particle emission at different times. Assuming chaoticity of the source, we derive a new general expression for the symmetrized two-particle cross section. We discuss the approximations needed to derive from the general result the Koonin-Pratt formula. Introducing a less stringent version of the so-called smoothness approximation we also derive a more accurate formula. It can be implemented into classical event generators and allows to calculate FSI corrected two-particle correlation functions via modified Bose-Einstein "weights".

I. INTRODUCTION

Two-particle correlations in momentum space can be used to extract information about the space-time structure of the emitting source [1,2]. The method exploits in an essential way the quantum mechanical uncertainty relation between coordinates and momenta, and thus any formal treatment of two-particle correlations must be based on a quantum mechanical description. For so-called "chaotic" sources where the two particles are emitted independently the description can be based on the *single-particle* Wigner density $S(x,K)$ of the source. It is, however, known to be important (at least in principle, but for sufficiently small sources also in practice) that one starts from the correct quantum mechanical Wigner density rather than directly from a classical space distribution $S_{\rm class}(x,K)$ of the particles in the source because the latter can lead to unphysical behavior of the correlation function [3]. The correlations are "generated" after the emission by two classes of effects:

(1) For pairs of identical bosons (fermions) the two-particle wave function describing their propagation towards the detector must be symmetrized (antisymmetrized). For boson pairs this results in quantum statistical "Bose-Einstein correlations" between the final state momenta of the two particles. Via the uncertainty relation these momentum space correlations reflect the spatial and temporal structure of the source from which the two particles were emitted. This correspondence forms the basis for Bose-Einstein interferometry in nuclear and particle physics [4], a variant of the well-known Hanbury Brown–Twiss intensity interferometry in astrophysics [5].

(2) For both pairs of identical and non-identical particles additional two-particle correlations can be generated by two-particle final state interactions, notably by the long-range Coulomb interaction if the particles carry charge [6–11]. These final state effects also depend on the spatial and temporal distance between the emission points of the two particles and thus must contain information about the space-time structure of the source. While in the last few years we have seen considerable progress in our understanding of how to extract from two-particle correlations quantitative information on both the geometric and the dynamic space-time structure (sizes, lifetime and expansion velocities) of the source *in the absence of final state interactions (FSI)* [2,13], not much is known about how to *quantitatively* correct these methods for FSI effects. This is largely due to the lack of a general and exact expression which relates the measured correlation function to the emission function of the source. In the absence of FSI (and for chaotic source) this relation reads [14–16,6]

$$C(\mathbf{q},\mathbf{K}) = 1 \pm \frac{\left| \int_x e^{iq\cdot x}\, S(x,K)\right|^2}{\int_x S\left(x, K{+}\frac{q}{2}\right) \int_y S\left(y, K{-}\frac{q}{2}\right)} , \qquad (1)$$

where $S(x,K)$ is the single particle Wigner density ("emission function") of the source, and where

$$\begin{aligned} \mathbf{K} &= \tfrac{1}{2}(\mathbf{p}_a + \mathbf{p}_b)\,, \quad & K^0 &= \tfrac{1}{2}(E_a + E_b)\,, \\ \mathbf{q} &= \mathbf{p}_a - \mathbf{p}_b\,, \quad & q^0 &= E_a - E_b = \mathbf{q}\cdot\mathbf{K}/K^0\,, \end{aligned} \qquad (2)$$

with $E_{a,b} = \sqrt{m^2 + \mathbf{p}_{a,b}^2}$. The second term in (1) reflects the quantum statistical correlations. This expression allows to expand the correlation function $C(\mathbf{q},\mathbf{K})$ in space-time moments of the emission function [13,17] and relate the main characteristics of the two-particle correlator, its width as a function of the relative momentum $\mathbf{q}$, to the second central space-time moment ("r.m.s. widths") of the emission function in coordinate space. The factor $e^{iq\cdot x}$ in the correlation term in (1) reflects the assumed absence of FSI, i.e. plane wave propagation.

The goal of the present paper is to generalize relation (1) to the case including two-particle FSI. Existing treatments of this problem [7–9,11,12] differ in their results even on the formal level because different types of approximations are used already in intermediate stages

of the calculation. This, unfortunately, makes a direct comparison of these results and a check of the approximations essentially impossible. Our aim is to give a rigorous derivation of the generalization of Eq. (1), using only the following two approximations:

(1) the source is completely chaotic, i.e. the particles are emitted independently, and

(2) finite multiplicity corrections [6,19] can be neglected.

Both approximations are expected to be good for high energy nuclear collisions with large multiplicities, but may require further scrutiny in lower multiplicity e^+e^- or hadron-hadron collisions. On the other hand, high multiplicities can result in modifications (screening) of the two-particle (Coulomb) potential due to the influence of the environment of other charged particles. It was, however, shown in [18] that such screening effects are small and can be neglected because the charge density of the environment drops rapidly as a function of the distance from the collision center (faster than $1/r^2$) and the pair leaves very quickly the region of high charge density.

II. THE TWO-PARTICLE CROSS SECTION

A. General strategy

The two-particle correlation function $C(\mathbf{p}_a, \mathbf{p}_b) \equiv C(\mathbf{q}, \mathbf{K})$ is defined as a ratio between the normalized invariant two-particle coincidence cross section $P_2(\mathbf{p}_a, \mathbf{p}_b)$ and the product of the invariant single particle cross sections $P_1(\mathbf{p}_{a,b})$:

$$C(\mathbf{q}, \mathbf{K}) = \frac{P_2(\mathbf{p}_a, \mathbf{p}_b)}{P_1(\mathbf{p}_a)\, P_1(\mathbf{p}_b)}\,, \tag{3}$$

where $P_2(\mathbf{p}_a, \mathbf{p}_b) = E_a E_b \frac{d^6 N}{d^3 p_a d^3 p_b}$, $P_1(\mathbf{p}_{a,b}) = E_{a,b} \frac{d^3 N}{d^3 p_{a,b}}$. Most space will be taken by the calculation of the two-particle emission probability $P_2(\mathbf{p}_a, \mathbf{p}_b)$. Its normalization by the product of single particle probabilities is a trivial final step.

Our strategy is as follows: We start by writing down an expression for the two-particle probability amplitude $A_\gamma(\mathbf{p}_a, \mathbf{p}_b)$ for measuring at $t \to \infty$ in the detector a pair of particles with momenta $\mathbf{p}_a$ and $\mathbf{p}_b$ if the pair has been emitted by the source in a two-particle state ψ_γ. The square of this amplitude, averaged properly over the distribution of two-particle quantum numbers γ in the source by taking the trace with an appropriate density matrix for the source, will give the two-particle probability $P_2(\mathbf{p}_a, \mathbf{p}_b)$. This amplitude, first written down in terms of an overlap integral at $t \to \infty$ with an asymptotic two-particle momentum eigenstate in the detector, is then rewritten in terms of a corresponding overlap integral in the source at the time of freeze-out, by using the time evolution operator (including the two-particle FSI) to evolve the asymptotic state backwards in time.

Here a crucial ingredient will be the realization that the two-particle FSI can only act while both particles are present. If one is emitted earlier than the other, in the absence of one-body final state interactions with the remainder of the source (e.g. between its charge and that of the remaining fireball which we will here neglect) it will undergo *free* time evolution until the second particle has also been formed. To implement this requires the factorization of the two-particle wave function into a product of single-particle states, i.e. the assumption of independent emission. This interval of free propagation gives rise to an additional phase which contains the time structure of the source and which may be important for attempts to extract source lifetimes from the two-particle correlator. As far as we know, this aspect has not been fully discussed in previous FSI studies [7–12].

Once the two-particle amplitude $A_\gamma(\mathbf{p}_a, \mathbf{p}_b)$ is expressed in terms of an overlap integral at the time of particle freeze-out, one can calculate the two-particle probability by averaging over the quantum numbers γ and the distribution of emission times. Due to the assumed factorization of the two-particle wave function the result can be expressed in terms of the single particle Wigner density of the source at freeze-out $S(x, K)$ which will be defined in Eq. (46). The remainder of the derivation consists of an appropriate rewriting of this expression which allows to compare it with Eq. (1) and which can serve as a starting point for further approximations in order to compare with previously published expressions in Refs. [7,8,11].

We should stress that our derivation is intrinsically non-relativistic. For this to be appropriate we must work in a reference frame in which the pair moves non-relativistically, i.e. where $\mathbf{K} \approx 0$. Since we are interested in the deviation of the correlator $C(\mathbf{q}, \mathbf{K})$ from unity for small $\mathbf{q}$, the relative motion is then also non-relativistic. In this frame the two-particle FSI can be represented by an instantaneous potential, and the difference between the emission times of the two particles is well-defined. The final result can then be evaluated in an arbitrary frame by proper Lorentz transformation of all momenta. A completely covariant derivation (which should also be possible by introducing propagating fields for the FSI) has not yet been achieved.

B. The two-particle momentum amplitude

Let us consider a two-particle state ψ_γ emitted by the source. Its propagation to the detector is governed by the Schrödinger equation

$$i \frac{\partial \psi_\gamma(\mathbf{x}_a, \mathbf{x}_b, t)}{\partial t} = \hat{H}(\mathbf{x}_a, \mathbf{x}_b)\, \psi_\gamma(\mathbf{x}_a, \mathbf{x}_b, t)\,, \tag{4}$$

where

$$\hat{H}(\mathbf{x}_a, \mathbf{x}_b) = \hat{h}(\mathbf{x}_a) + \hat{h}(\mathbf{x}_b) + V\left(|\mathbf{x}_a - \mathbf{x}_b|\right)\,, \tag{5}$$

and

$$\hat{h}(\mathbf{x}_i) = m - \frac{1}{2m}\boldsymbol{\nabla}_i^2 . \tag{6}$$

The index γ denotes a complete set of 2-particle quantum numbers. (In a basis of products of two wave packets these could contain the centers $\mathbf{X}_a$, $\mathbf{X}_b$ of the wavepackets of the two particles at their freeze-out times t_a, t_b, respectively.) Eq. (4) is solved by

$$\psi_\gamma(\mathbf{x}_a,\mathbf{x}_b,t) = e^{-i\hat{H}(\mathbf{x}_a,\mathbf{x}_b)(t-t_0)}\,\psi_\gamma(\mathbf{x}_a,\mathbf{x}_b,t_0) \tag{7}$$

in terms of the two-particle wave function at some initial time t_0. We will assume that the detector measures asymptotic momentum eigenstates, i.e. that it acts by projecting the emitted 2-particle state onto

$$\begin{aligned}\phi^{\rm out}_{\mathbf{p}_a,\mathbf{p}_b}(\mathbf{x}_a,\mathbf{x}_b,t) &= e^{i(\mathbf{p}_a\cdot\mathbf{x}_a - E_a t)}\,e^{i(\mathbf{p}_b\cdot\mathbf{x}_b - E_b t)}\\ &= e^{-iEt}\,e^{i(\mathbf{p}_a\cdot\mathbf{x}_a+\mathbf{p}_b\cdot\mathbf{x}_b)},\end{aligned} \tag{8}$$

where $E = E_a + E_b = 2K^0$ with $E_{a,b} = \sqrt{m^2+\mathbf{p}_{a,b}^2}$ is the total energy of the pair. In this paper we will only consider the case of pairs of identical particles, $m_a = m_b = m$; correspondingly, the two-particle states ψ_γ in (7) must be (anti-)symmetrized (see below). The measured two-particle momentum amplitude is then

$$\begin{aligned}&A_\gamma(\mathbf{p}_a,\mathbf{p}_b) =\\ &\lim_{t\to\infty}\int d^3x_a\,d^3x_b\,\phi^{{\rm out},*}_{\mathbf{p}_a,\mathbf{p}_b}(\mathbf{x}_a,\mathbf{x}_b,t)\,\psi_\gamma(\mathbf{x}_a,\mathbf{x}_b,t).\end{aligned} \tag{9}$$

Using the time evolution equation (7) this can be expressed in terms of the emitted two-particle wave function ψ_γ at earlier times as

$$\begin{aligned}A_\gamma(\mathbf{p}_a,\mathbf{p}_b) &= \lim_{t\to\infty}\int d^3x_a\,d^3x_b\,\psi_\gamma(\mathbf{x}_a,\mathbf{x}_b,t_0)\\ &\times\Big[\exp[-i\hat{H}(\mathbf{x}_a,\mathbf{x}_b)(t_0-t)]\,\phi^{\rm out}_{\mathbf{p}_a,\mathbf{p}_b}(\mathbf{x}_a,\mathbf{x}_b,t)\Big]^* .\end{aligned} \tag{10}$$

This is correct for all times $t_0 \geq \max\,[t_a,t_b]$ where $t_{a,b}$ are the freeze-out times for the two particles. Note that by partial integration (or inversion of the unitary evolution operator) the time evolution operator has been shifted from the emitted two-particle state (with arbitrary quantum numbers γ) to the two-particle momentum eigenstate $\phi^{\rm out}_{\mathbf{p}_a,\mathbf{p}_b}(\mathbf{x}_a,\mathbf{x}_b,t)$ which thereby is transformed into a distorted wave at time t_0 which includes the effect of the FSI in such a way that after evolution from t_0 to $t=\infty$ it again becomes a plane wave with momentum $\mathbf{p}_a$, $\mathbf{p}_b$ and energy $E = E_a + E_b$.

C. The distorted wave

To further analyze Eq. (10) we introduce center-of-mass and relative coordinates according to

$$\begin{aligned}\mathbf{R} &= \frac{1}{2}(\mathbf{x}_a+\mathbf{x}_b), \quad & \mathbf{P} &= 2\mathbf{K} = \mathbf{p}_a+\mathbf{p}_b,\\ \mathbf{r} &= \mathbf{x}_a-\mathbf{x}_b, & \mathbf{q} &= \mathbf{p}_a-\mathbf{p}_b,\end{aligned} \tag{11}$$

such that

$$\boldsymbol{\nabla}_a^2 + \boldsymbol{\nabla}_b^2 = \frac{1}{2}\boldsymbol{\nabla}_R^2 + 2\boldsymbol{\nabla}_r^2, \tag{12}$$

and

$$\begin{aligned}\hat{H}(\mathbf{x}_a,\mathbf{x}_b) &= \hat{H}_0(\mathbf{R}) + \hat{H}_1(\mathbf{r}),\\ \hat{H}_0(\mathbf{R}) &\equiv -\frac{1}{2M}\boldsymbol{\nabla}_R^2,\\ \hat{H}_1(\mathbf{r}) &\equiv -\frac{1}{2\mu}\boldsymbol{\nabla}_r^2 + V(r),\end{aligned} \tag{13}$$

with $M = 2m$ and $\mu = m/2$ for identical particles. In these coordinates the asymptotic wave function (8) reads

$$\phi^{\rm out}_{\mathbf{p}_a,\mathbf{p}_b}(\mathbf{x}_a,\mathbf{x}_b,t) = e^{i(\mathbf{P}\cdot\mathbf{R}-Et)}\,e^{\frac{i}{2}\mathbf{q}\cdot\mathbf{r}}. \tag{14}$$

In Eq. (10) we must evaluate the action of the time evolution operator on this function. To this end let us separate the total energy $E = 2K^0$ into its contributions from the c.m. and relative motions, $E = E_{\rm cm} + E_{\rm rel}$, where $E_{\rm cm} = \sqrt{M^2+\mathbf{P}^2} = 2\sqrt{m^2+\mathbf{K}^2} \equiv 2E_K \approx M + \mathbf{P}^2/2M$ and $E_{\rm rel} = E - E_{\rm cm} = 2(K^0 - E_K) \approx \mathbf{q}^2/(4E_K) \approx \mathbf{q}^2/4m$ in a frame where $\mathbf{K}\approx 0$. The action of $\hat{H}_0(\mathbf{R})$ on the plane wave factor for the c.m. motion is then easily evaluated:

$$e^{-i\hat{H}_0(\mathbf{R})(t_0-t)}\,e^{i\mathbf{P}\cdot\mathbf{R}} = e^{-i\sqrt{M^2+\mathbf{P}^2}\,(t_0-t)}\,e^{i\mathbf{P}\cdot\mathbf{R}}. \tag{15}$$

What is left is the time evolution of the wave function for the relative motion:

$$\lim_{t\to\infty} e^{-i\hat{H}_1(\mathbf{r})(t_0-t)}\,e^{\frac{i}{2}\mathbf{q}\cdot\mathbf{r}}\,e^{-iE_{\rm rel}t} \equiv e^{-iE_{\rm rel}\,t_0}\,\phi_{\mathbf{q}/2}(\mathbf{r}). \tag{16}$$

Here $\phi_{\mathbf{q}/2}(\mathbf{r})$ is a solution of the stationary Schrödinger equation

$$\hat{H}_1(\mathbf{r})\,\phi_{\mathbf{q}/2}(\mathbf{r}) = E_{\rm rel}\phi_{\mathbf{q}/2}(\mathbf{r}), \tag{17}$$

(where $E_{\rm rel} = \frac{\mathbf{q}^2}{4E_K}$) with asymptotic boundary conditions

$$\lim_{|\mathbf{r}|\to\infty}\phi_{\mathbf{q}/2}(\mathbf{r}) = e^{\frac{i}{2}\mathbf{q}\cdot\mathbf{r}}. \tag{18}$$

Inserting everything into Eq. (10) we find

$$\begin{aligned}A_\gamma(\mathbf{p}_a,\mathbf{p}_b) = e^{iEt_0}\int d^3r\,d^3R\,e^{-i\mathbf{P}\cdot\mathbf{R}}\phi^*_{\mathbf{q}/2}(\mathbf{r})\\ \times\,\psi_\gamma\left(\mathbf{R}+\tfrac{\mathbf{r}}{2},\mathbf{R}-\tfrac{\mathbf{r}}{2},t_0\right).\end{aligned} \tag{19}$$

D. Free propagation between emission points

With Eq. (19) we have rewritten the measured two-particle momentum space amplitude $A_\gamma(\mathbf{p}_a, \mathbf{p}_b)$ in terms of an overlap integral evaluated at the earliest time t_0 where both particles are present. Let us, for the moment, call this time t_a. At this time the first emitted particle of the pair has already propagated for a time $t_a - t_b$ if it was emitted at $t_b < t_a$. During this time the first emitted particle cannot "see" the second particle as a separate entity, but only as part of the remaining fireball. Nevertheless, its charge is there, "hidden" among the charges of all the other particles in the fireball.

This stage of the time evolution is obviously very complicated. The idealization of letting the two particles interact via a two-body (Coulomb) potential clearly breaks down, since the second particle has not been "formed" yet. On the other hand, considering the first emitted particle as freely propagating during this time is not necessarily a good approximation either: there should at least be the interaction with charge of the rest of the fireball which during this time interval is one unit larger than after emission of the second particle. However, considering both the interaction between the two particles and between each one of them and the remaining fireball is a non-trivial quantum mechanical tree-body problem, at least until the pair has well separated from the fireball.

We will therefore study here only the simple approximation that the first emitted particle propagates freely until the second particle freezes out. In order to implement this into the formalism we must make the assumption that the two particles are emitted independently, i.e. that at t_a (up to symmetrization effects) the two-particle wave function ψ_γ factorizes:

$$\psi_\gamma(\mathbf{x}_a, \mathbf{x}_b, t_a) = \frac{1}{\sqrt{2}}[\psi_{\gamma_a}(\mathbf{x}_a, t_a)\, \psi_{\gamma_b}(\mathbf{x}_b, t_a) \pm \psi_{\gamma_a}(\mathbf{x}_b, t_a)\, \psi_{\gamma_b}(\mathbf{x}_a, t_a)] \,. \tag{20}$$

The indices γ_a, γ_b on the 1-particle wave functions now label complete sets of 1-particle quantum numbers. For simplicity of notation we will from now on replace $\gamma_{a,b}$ by a, b.

Let us first consider the case that the particle in the state ψ_b was emitted earlier, at time $t_b < t_a$. We can then write (see Eq. (5))

$$\psi_b(\mathbf{x}, t_a) = e^{-i\hat{h}(\mathbf{x})(t_a - t_b)} \psi_b(\mathbf{x}, t_b) \,, \tag{21}$$

and hence

$$\psi_\gamma(\mathbf{x}_a, \mathbf{x}_b, t_a) = \frac{\theta(t_a - t_b)}{\sqrt{2}} \Big[\psi_a(\mathbf{x}_a, t_a)\, e^{-i\hat{h}(\mathbf{x}_b)(t_a - t_b)} \psi_b(\mathbf{x}_b, t_b) \pm \psi_a(\mathbf{x}_b, t_a)\, e^{-i\hat{h}(\mathbf{x}_a)(t_a - t_b)} \psi_b(\mathbf{x}_a, t_b) \Big] \,. \tag{22}$$

In order to evaluate the action of the free time-evolution operator we Fourier decompose ψ_a, ψ_b into momentum eigenstates:

$$\psi_a(\mathbf{x}, t) = \int \frac{d^3k_a}{(2\pi)^3}\, \tilde{\psi}_a(\mathbf{k}_a, t)\, e^{i(\mathbf{k}_a \cdot \mathbf{x} - \omega_a t)} \,, \tag{23a}$$

$$\omega_a = \sqrt{m^2 + \mathbf{k}_a^2} \,, \tag{23b}$$

and similarly for ψ_b with integration variable $\mathbf{k}_b$. One then obtains

$$\psi_\gamma(\mathbf{x}_a, \mathbf{x}_b, t_a) = \frac{\theta(t_a - t_b)}{\sqrt{2}} \int \frac{d^3k_a}{(2\pi)^3} \frac{d^3k_b}{(2\pi)^3} \times \tilde{\psi}_a(\mathbf{k}_a, t_a)\, \tilde{\psi}_b(\mathbf{k}_b, t_b)\; e^{-i[\omega_a t_a + \omega_b(t_a - t_b) + \omega_b t_b]} \times \left[e^{i(\mathbf{k}_a \cdot \mathbf{x}_a + \mathbf{k}_b \cdot \mathbf{x}_b)} \pm e^{i(\mathbf{k}_b \cdot \mathbf{x}_a + \mathbf{k}_a \cdot \mathbf{x}_b)} \right] , \tag{24}$$

where we have written out in the time factor separately the contributions from the Fourier transform and from the free time evolution according to (22). Note that the terms $\sim t_b$ cancel between them.

Inserting (24) into (19) and defining

$$\mathbf{Q} = \mathbf{k}_a + \mathbf{k}_b \,, \quad \mathbf{k} = \frac{1}{2}(\mathbf{k}_a - \mathbf{k}_b) \tag{25}$$

we arrive at

$$A^{(a)}_{ab}(\mathbf{p}_a, \mathbf{p}_b) = \frac{\theta(t_a - t_b)}{\sqrt{2}} \int \frac{d^3Q}{(2\pi)^3} \frac{d^3k}{(2\pi)^3}\, d^3r\, d^3R \times e^{i(\mathbf{Q}-\mathbf{P})\cdot\mathbf{R}}\, e^{i\mathbf{k}\cdot\mathbf{r}}\, e^{i\Omega(\mathbf{Q},\mathbf{k})t_a} \left[\phi^*_{\mathbf{q}/2}(\mathbf{r}) \pm \phi^*_{-\mathbf{q}/2}(\mathbf{r}) \right] \times \tilde{\psi}_a\left(\tfrac{\mathbf{Q}}{2} + \mathbf{k}, t_a\right) \tilde{\psi}_b\left(\tfrac{\mathbf{Q}}{2} - \mathbf{k}, t_b\right) . \tag{26}$$

The superscript (a) in the amplitude should remind us that the appearance of the second particle (i.e. the creation of the "pair") happened at time t_a. In deriving (26) we used the fact that both $\phi_{\mathbf{q}/2}(\mathbf{r})$ and $\phi_{-\mathbf{q}/2}(\mathbf{r})$ are solutions to (17) with the same energy eigenvalue, but corresponding to boundary conditions (18) with opposite signs in the exponent. From this it follows immediately that

$$\phi_{-\mathbf{q}/2}(\mathbf{r}) = \phi_{\mathbf{q}/2}(-\mathbf{r}) \,. \tag{27}$$

With this trick the symmetrization effects on the emitted two-particle state can be absorbed into the relative wavefunction describing the FSI. – In (26) we also replaced the two-particle quantum number index γ by the pair (ab) of single particle quantum numbers and defined

$$\Omega\,(\mathbf{Q}, \mathbf{k}) \equiv E - \omega\left(\tfrac{\mathbf{Q}}{2} + \mathbf{k}\right) - \omega\left(\tfrac{\mathbf{Q}}{2} - \mathbf{k}\right) . \tag{28}$$

The integration over the c.m. coordinate $\mathbf{R}$ of the pair is trivial and yields $(2\pi)^3\, \delta^3(\mathbf{Q} - \mathbf{P})$. After doing the integrations over $\mathbf{Q}$ and $\mathbf{r}$ we get (remember that $\mathbf{P} = 2\mathbf{K} = \mathbf{p}_a + \mathbf{p}_b$)

$$A^{(a)}_{ab}(\mathbf{p}_a, \mathbf{p}_b) = \theta(t_a - t_b) \int \frac{d^3k}{(2\pi)^3}\, e^{i\Omega(\mathbf{K},\mathbf{k})t_a}\, \tilde{\Phi}^*\left(\tfrac{\mathbf{q}}{2}, \mathbf{k}\right) \times \tilde{\psi}_a(\mathbf{K} + \mathbf{k}, t_a)\, \tilde{\psi}_b(\mathbf{K} - \mathbf{k}, t_b) \,. \tag{29}$$

Here

$$\Omega(\mathbf{K},\mathbf{k}) \equiv E_a - \omega(\mathbf{K}+\mathbf{k}) + E_b - \omega(\mathbf{K}-\mathbf{k})\,, \qquad (30)$$

and we defined the symmetrized FSI distorted wave

$$\Phi\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) = \frac{1}{\sqrt{2}}\left[\tilde{\phi}\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) \pm \tilde{\phi}\left(-\tfrac{\mathbf{q}}{2},\mathbf{k}\right)\right]\,, \qquad (31)$$

where

$$\tilde{\phi}\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) = \int d^3r\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, \phi_{\mathbf{q}/2}(\mathbf{r}) \qquad (32)$$

is the momentum space representation of the distorted relative wave function with asymptotic relative momentum $\mathbf{q}/2$.

The opposite situation that the particle in the state ψ_a was emitted earlier at time t_a and the "pair" appears only later at time t_b can be dealt with in a similar way. One finds

$$A^{(b)}_{ab}(\mathbf{p}_a,\mathbf{p}_b) = \theta(t_b - t_a)\int \frac{d^3k}{(2\pi)^3}\, e^{i\Omega(\mathbf{K},\mathbf{k})t_b}\, \Phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) \times \tilde{\psi}_a(\mathbf{K}+\mathbf{k},t_a)\,\tilde{\psi}_b(\mathbf{K}-\mathbf{k},t_b)\,. \qquad (33)$$

The total amplitude is the sum of the contributions from the two different time orderings:

$$A_{ab}(\mathbf{p}_a,\mathbf{p}_b) = A^{(a)}_{ab}(\mathbf{p}_a,\mathbf{p}_b) + A^{(b)}_{ab}(\mathbf{p}_a,\mathbf{p}_b)\,. \qquad (34)$$

Let us note for later reference that neglecting the free time evolution of the first emitted particle from time t_b to time t_a would have resulted in the ansatz

$$\psi_\gamma(\mathbf{x}_a,\mathbf{x}_b,t_a) = \frac{\theta(t_a-t_b)}{\sqrt{2}} \times\Big[\psi_a(\mathbf{x}_a,t_a)\,\psi_b(\mathbf{x}_b,t_b) \pm \psi_a(\mathbf{x}_b,t_a)\,\psi_b(\mathbf{x}_a,t_b)\Big] \qquad (35)$$

instead of (22) for the time ordering (a), and similarly with $\theta(t_a-t_b)$ replaced by $\theta(t_b-t_a)$ for the time ordering (b). This would have resulted in the modified amplitudes

$$A^{(a)}_{ab}(\mathbf{p}_a,\mathbf{p}_b) = \theta(t_a-t_b)\, e^{iEt_a}\int \frac{d^3k}{(2\pi)^3}\,\Phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) \times e^{-i\omega(\mathbf{K}+\mathbf{k})t_a}\,\tilde{\psi}_a(\mathbf{K}+\mathbf{k},t_a) \times e^{-i\omega(\mathbf{K}-\mathbf{k})t_b}\,\tilde{\psi}_b(\mathbf{K}-\mathbf{k},t_b)\,, \qquad (36a)$$

$$A^{(b)}_{ab}(\mathbf{p}_a,\mathbf{p}_b) = \theta(t_b-t_a)\, e^{iEt_b}\int \frac{d^3k}{(2\pi)^3}\,\Phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) \times e^{-i\omega(\mathbf{K}+\mathbf{k})t_a}\,\tilde{\psi}_a(\mathbf{K}+\mathbf{k},t_a) \times e^{-i\omega(\mathbf{K}-\mathbf{k})t_b}\,\tilde{\psi}_b(\mathbf{K}-\mathbf{k},t_b)\,. \qquad (36b)$$

E. The two-particle cross-section

The two-particle cross section is obtained by averaging (34) and its complex conjugate with the density matrix defining the source. This density matrix is characterized by a probability distribution for the two-particle quantum numbers (a,b) and by a distribution of emission times $(t_a,\, t_b)$. We write

$$P_2(\mathbf{p}_a,\mathbf{p}_b) = \sum\!\!\!\!\!\!\int_{ab,a'b'} \rho_{ab,a'b'}\, A^*_{a'b'}(\mathbf{p}_a,\mathbf{p}_b)\, A_{ab}(\mathbf{p}_a,\mathbf{p}_b)\,, \qquad (37)$$

and make the ansatz

$$\rho_{ab,a'b'} = \nu_{aa'}\,\rho(t_a,t_{a'})\,\nu_{bb'}\,\rho(t_b,t_{b'})\,. \qquad (38)$$

This ansatz factorizes in such a way that independent emission of the two particles is ensured. The summation/integration in (37) is to be understood as

$$\sum\!\!\!\!\!\!\int_{ab,a'b'} = \sum_{ab,a'b'}\int dt_a\, dt_b\, dt_{a'}\, dt_{b'}\,. \qquad (39)$$

According to (34) the probability consists of four terms which we write as

$$P_2(\mathbf{p}_a,\mathbf{p}_b) = P^{(aa)} + P^{(bb)} + P^{(ab)} + P^{(ba)}\,. \qquad (40)$$

We shall calculate only the first term $P^{(aa)}$ explicitly. The second term is easily shown to equal the first one while the last two terms will be shown to vanish.

Inserting (29) into (37) yields

$$P^{(aa)}(\mathbf{p}_a,\mathbf{p}_b) = \int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\,\Phi\left(\tfrac{\mathbf{q}}{2},\mathbf{k}'\right)\Phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) \times \int dt_a dt_b dt_{a'} dt_{b'}\,\theta(t_a-t_b)\,\theta(t_{a'}-t_{b'}) \times e^{i(\Omega(\mathbf{K},\mathbf{k})t_a - \Omega(\mathbf{K},\mathbf{k}')t_{a'})} \times \sum_{aa'}\nu_{aa'}\,\rho(t_a,t_{a'})\,\tilde{\psi}_a(\mathbf{K}+\mathbf{k},t_a)\,\tilde{\psi}^*_{a'}(\mathbf{K}+\mathbf{k}',t_{a'}) \times \sum_{bb'}\nu_{bb'}\,\rho(t_b,t_{b'})\,\tilde{\psi}_b(\mathbf{K}-\mathbf{k},t_b)\,\tilde{\psi}^*_{b'}(\mathbf{K}-\mathbf{k}',t_{b'}). \qquad (41)$$

Let us introduce new time variables

$$X^0 = \frac{1}{2}(t_a + t_{a'})\,, \qquad x^0 = t_a - t_{a'}\,,$$
$$Y^0 = \frac{1}{2}(t_b + t_{b'})\,, \qquad y^0 = t_b - t_{b'}\,, \qquad (42)$$

as well as the inverse Fourier representation (see Eq. (23))

$$\tilde{\psi}_a\left(\mathbf{K}+\mathbf{k}, X^0 + \tfrac{x^0}{2}\right)\tilde{\psi}^*_{a'}\left(\mathbf{K}+\mathbf{k}', X^0 - \tfrac{x^0}{2}\right) = \qquad (43a)$$
$$\int d^3X\, d^3x\; \psi_a\left(X + \tfrac{x}{2}\right) e^{i\left[\omega(\mathbf{K}+\mathbf{k})\left(X^0+\frac{1}{2}x^0\right) - (\mathbf{K}+\mathbf{k})\cdot\left(\mathbf{X}+\frac{1}{2}\mathbf{x}\right)\right]}\; \psi^*_{a'}\left(X - \tfrac{x}{2}\right) e^{-i\left[\omega(\mathbf{K}+\mathbf{k}')\left(X^0-\frac{1}{2}x^0\right) - (\mathbf{K}+\mathbf{k}')\cdot\left(\mathbf{X}-\frac{1}{2}\mathbf{x}\right)\right]}\,,$$

$$\tilde{\psi}_b\left(\mathbf{K}-\mathbf{k}, Y^0+\tfrac{y^0}{2}\right)\,\tilde{\psi}^*_{b'}\left(\mathbf{K}-\mathbf{k}', Y^0-\tfrac{y^0}{2}\right) = \tag{43b}$$
$$\int d^3Y\, d^3y$$
$$\psi_b\left(Y+\tfrac{y}{2}\right)\, e^{i\left[\omega(\mathbf{K}-\mathbf{k})\left(Y^0+\frac{1}{2}y^0\right)-(\mathbf{K}-\mathbf{k})\cdot\left(\mathbf{Y}+\frac{1}{2}\mathbf{y}\right)\right]}$$
$$\psi^*_{b'}\left(Y-\tfrac{y}{2}\right)\, e^{-i\left[\omega(\mathbf{K}-\mathbf{k}')\left(Y^0-\frac{1}{2}y^0\right)-(\mathbf{K}-\mathbf{k}')\cdot\left(\mathbf{Y}-\frac{1}{2}\mathbf{y}\right)\right]}\,.$$

The phase in (41) reduces to

$$\Omega(\mathbf{K},\mathbf{k})t_a - \Omega(\mathbf{K},\mathbf{k}')t_{a'} = \tag{44}$$
$$X^0\left[\omega(\mathbf{K}+\mathbf{k}')-\omega(\mathbf{K}+\mathbf{k})+\omega(\mathbf{K}-\mathbf{k}')-\omega(\mathbf{K}-\mathbf{k})\right]$$
$$+x^0\Big[2K^0$$
$$-\tfrac{1}{2}\left[\omega(\mathbf{K}+\mathbf{k}')+\omega(\mathbf{K}+\mathbf{k})+\omega(\mathbf{K}-\mathbf{k}')+\omega(\mathbf{K}-\mathbf{k})\right]\Big]\,,$$

while the θ-functions become

$$\theta\left(X^0-Y^0+\tfrac{x^0-y^0}{2}\right)\theta\left(X^0-Y^0-\tfrac{x^0-y^0}{2}\right)$$
$$=\int\frac{d\omega}{2\pi}\,\frac{1}{\omega+i\epsilon}\,e^{-i\omega\left[X^0-Y^0+\frac{1}{2}(x^0-y^0)\right]}$$
$$\times\int\frac{d\omega'}{2\pi}\,\frac{1}{\omega'-i\epsilon}\,e^{i\omega'\left[X^0-Y^0-\frac{1}{2}(x^0-y^0)\right]}\,. \tag{45}$$

We now define the single particle Wigner density $S(X,K)$ of the source as

$$S(X,K)=\int d^4x\, e^{iK\cdot x}\,\rho\left(X^0+\tfrac{x^0}{2},X^0-\tfrac{x^0}{2}\right)$$
$$\times\sum_{aa'}\nu_{aa'}\,\psi_a\left(X+\tfrac{x}{2}\right)\,\psi^*_{a'}\left(X-\tfrac{x}{2}\right)\,. \tag{46}$$

Using the hermiticity of the density matrix one easily shows that $S(X,K)$ real. Then we can combine Eqs. (43)-(46) to rewrite (41) as

$$P^{(aa)}(\mathbf{p}_a,\mathbf{p}_b)=\int\frac{d^4k}{(2\pi)^4}\,\frac{d^4k'}{(2\pi)^4}\,\Phi\left(\tfrac{\mathbf{q}}{2},\mathbf{k}'\right)\,\Phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right)$$
$$\times\frac{1}{K^0-k^0-\omega(\mathbf{K}-\mathbf{k})+i\epsilon}\,\frac{1}{K^0-k^{0\prime}-\omega(\mathbf{K}-\mathbf{k}')-i\epsilon}$$
$$\times\int d^4X\,d^4Y\,e^{i(k-k')\cdot(X-Y)}$$
$$\times\, S\left(X,K+\tfrac{k+k'}{2}\right)\,S\left(Y,K-\tfrac{k+k'}{2}\right)\,. \tag{47a}$$

To obtain this expression we shifted the integration variables in (45) by defining $k^0=K^0-\omega-\omega(\mathbf{K}-\mathbf{k})$, $k^{0\prime}=K^0-\omega'-\omega(\mathbf{K}-\mathbf{k}')$.

The second diagonal term $P^{(bb)}$ can be calculated similarly:

$$P^{(bb)}(\mathbf{p}_a,\mathbf{p}_b)=\int\frac{d^4k}{(2\pi)^4}\,\frac{d^4k'}{(2\pi)^4}\,\Phi\left(\tfrac{\mathbf{q}}{2},\mathbf{k}'\right)\,\Phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right)$$
$$\times\frac{1}{K^0+k^0-\omega(\mathbf{K}+\mathbf{k})+i\epsilon}\,\frac{1}{K^0+k^{0\prime}-\omega(\mathbf{K}+\mathbf{k}')-i\epsilon}$$
$$\times\int d^4X\,d^4Y\,e^{i(k-k')\cdot(X-Y)}$$
$$\times\, S\left(X,K+\tfrac{k+k'}{2}\right)\,S\left(Y,K-\tfrac{k+k'}{2}\right)\,. \tag{47b}$$

By relabelling the integration variables,

$$X\rightleftharpoons Y \quad\text{and}\quad k\to -k\,,\quad k'\to -k'\,,$$

and taking into account that $\Phi\left(\frac{1}{2}\mathbf{q},-\mathbf{k}\right)=\pm\Phi\left(\frac{1}{2}\mathbf{q},\mathbf{k}\right)$ for bosons and fermions, respectively, one easily checks the equality

$$P^{(aa)}(\mathbf{p}_a,\mathbf{p}_b)=P^{(bb)}(\mathbf{p}_a,\mathbf{p}_b)\,. \tag{47c}$$

The cross terms $P^{(ab)}$ and $P^{(ba)}$ represent interference between amplitudes of opposite time ordering. They vanish by causality. Indeed, each cross term contains under the integral the product of two retarded propagators, e.g. $1/[K^0-k^0-\omega(\mathbf{K}-\mathbf{k})+i\epsilon]$ and $1/[K^0+k^{0\prime}-\omega(\mathbf{K}+\mathbf{k}')-i\epsilon]$ in $P^{(ab)}$, which in the complex k^0 and $k^{0\prime}$ planes have poles on the same side of the real k^0 and $k^{0\prime}$ axes while the corresponding time-energy exponents in the plane wave factor have opposite signs. This latter fact means that when doing the k^0 and $k^{0\prime}$ integrations, the corresponding contours must be closed in opposite half planes, thus always missing one of the poles.

Altogether we thus find

$$P_2(\mathbf{p}_a,\mathbf{p}_b)=2P^{(aa)}(\mathbf{p}_a,\mathbf{p}_b)\,. \tag{47d}$$

Equations (47) are the main result of this paper. In the following section we discuss it further and study various approximations. At this point let us only note that if we neglect the phase factor resulting from the free propagation of the first emitted particle until emission of the second one, i.e. use Eq. (36) for the two-particle momentum amplitude, we obtain a very similar expression to (47a) where only the terms $\omega(\mathbf{K}-\mathbf{k})$ resp. $-\omega(\mathbf{K}-\mathbf{k}')$ in the two energy denominators are missing. The consequences will be discussed below.

III. DISCUSSION AND APPROXIMATIONS

Expression (47a) for the two-particle spectrum is not very convenient in practice, because of the poles resulting from the two energy denominators. Their contribution can, however, be evaluated analytically by doing the k^0, $k^{0\prime}$ integrations using residue calculus. For $X^0-Y^0>0$ one can close the two integration contours in such a way that both poles are encircled and contribute. One then obtains

$$P_2(\mathbf{p}_a,\mathbf{p}_b)=2\int\frac{d^3k}{(2\pi)^3}\,\frac{d^3k'}{(2\pi)^3}\,\Phi\left(\tfrac{\mathbf{q}}{2},\mathbf{k}'\right)\,\Phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right)$$
$$\int d^4X\,d^4Y\,\theta(X^0-Y^0)\,e^{i(k-k')\cdot(X-Y)}$$
$$\times\, S\left(X,K+\tfrac{k+k'}{2}\right)\,S\left(Y,K-\tfrac{k+k'}{2}\right)\,, \tag{48a}$$

where

$$k^0=K^0-\omega(\mathbf{K}-\mathbf{k})\,,\quad k^{0\prime}=K^0-\omega(\mathbf{K}-\mathbf{k}') \tag{48b}$$

if the free time evolution between emission points is included, while

$$k^0 = k^{0\prime} = K^0 \ , \tag{48c}$$

if it is neglected.

A. Wigner representation

Equation (47) can be written as a folding relation between Wigner densities. We define $p = \frac{1}{2}(k + k')$, $Q = k - k'$, and

$$\chi_{\mathbf{q}/2}(p) \equiv \Phi\left(\tfrac{\mathbf{q}}{2}, \mathbf{p}\right) \, \frac{1}{p^0 - K^0 + \omega(\mathbf{p} - \mathbf{K}) + i\epsilon} \tag{49a}$$

for the case where the propagation between emission points is included, or

$$\chi_{\mathbf{q}/2}(p) \equiv \Phi\left(\tfrac{\mathbf{q}}{2}, \mathbf{p}\right) \, \frac{1}{p^0 - K^0 + i\epsilon} \tag{49b}$$

if free propagation between emission points is neglected. The two-particle cross section (47) then reads

$$\begin{aligned} P_2(\mathbf{p}_a, \mathbf{p}_b) = \int \frac{d^4p}{(2\pi)^4} \, d^4X \, d^4Y \, S(X, K+p) \\ \times \, W_{\mathbf{q}/2}(X-Y, p) \, S(Y, K-p) \, , \end{aligned} \tag{50}$$

with

$$\begin{aligned} W_{\mathbf{q}/2}(X-Y, p) = \int \frac{d^4Q}{(2\pi)^4} \, e^{-iQ\cdot(X-Y)} \\ \times \, \chi_{\mathbf{q}/2}\left(p + \tfrac{Q}{2}\right) \, \chi^*_{\mathbf{q}/2}\left(p - \tfrac{Q}{2}\right) \, . \end{aligned} \tag{51}$$

This function W can be interpreted as the Wigner density associated with the distorted wave describing the final state interactions. It describes the power of the FSI to "push" two particles, which were originally emitted with momenta $p'_a = K + p$ and $p'_b = K - p$, to the observed values p_a and p_b (resp. K and q). It is determined by the "modified distorted waves" χ. Please note that the representation (50) is generic; it was previously derived by Pratt in Eq. (2.8) of Ref. [11] with quite different methods. Different approximations in dealing with the propagation of the first emitted particle between the emission points only result in different "modification factors" associated with the distorted waves $\Phi\left(\frac{\mathbf{q}}{2}, \mathbf{p}\right)$. Equations (49a) and (49b) are two specific examples: we expect that different assumptions about what happens to the first particle while it waits for the appearance of the second one can be similarly included in Eq. (51) by simply changing the "modification factor" in the definition of $\chi_{\mathbf{q}/2}(p)$.

B. Free particle limit

To discuss the limiting case of no final state interactions it is best to start from Eq. (48a). In this case the wave functions $\phi_{\pm\mathbf{q}/2}(\mathbf{r})$ in (17) describing the relative motion of the two particles on their way to the detector become plane waves, with the momentum space representation

$$\Phi\left(\frac{\mathbf{q}}{2}, \mathbf{k}\right) = \frac{(2\pi)^3}{\sqrt{2}} \left[\delta^{(3)}\left(\tfrac{\mathbf{q}}{2} - \mathbf{k}\right) \pm \delta^{(3)}\left(\tfrac{\mathbf{q}}{2} + \mathbf{k}\right)\right] . \tag{52}$$

The two Φ-functions in (48a) result in four terms two of which correspond to $\frac{1}{2}(\mathbf{k}+\mathbf{k}') = \pm\mathbf{q}$, $\mathbf{k}-\mathbf{k}' = 0$, while the other two terms have $\frac{1}{2}(\mathbf{k}+\mathbf{k}') = 0$, $\mathbf{k}-\mathbf{k}' = \pm\mathbf{q}$. The corresponding energy values depend on whether or not we include free propagation between the emission points. If we include it, the energies are $\frac{1}{2}(k^0 + k^{0\prime}) = \pm q^0$, $k^0 - k^{0\prime} = 0$ for the first two terms and $\frac{1}{2}(k^0 + k^{0\prime}) = 0$, $k^0 - k^{0\prime} = \pm q^0$ for the last two terms ($q^0 = E_a - E_b$). The two-particle cross section becomes

$$\begin{aligned} P_2(\mathbf{p}_a, \mathbf{p}_b) = \int d^4X \, d^4Y \, \theta(X^0 - Y^0) \\ \times \Big\{ S\left(X, K+\tfrac{q}{2}\right) \, S\left(Y, K-\tfrac{q}{2}\right) \\ + S\left(X, K-\tfrac{q}{2}\right) \, S\left(Y, K+\tfrac{q}{2}\right) \\ \pm \left[e^{iq\cdot(X-Y)} + e^{-iq\cdot(X-Y)}\right] S(X,K) \, S(Y,K) \Big\} . \end{aligned} \tag{53}$$

After relabelling $X \leftrightarrow Y$ in the second and fourth term and using $\theta(X^0 - Y^0) + \theta(Y^0 - X^0) = 1$ we obtain

$$\begin{aligned} P_2(\mathbf{p}_a, \mathbf{p}_b) = \int d^4X \, d^4Y \Big[S\left(X, K+\tfrac{q}{2}\right) \, S\left(Y, K-\tfrac{q}{2}\right) \\ \pm e^{iq\cdot(X-Y)} \, S(X,K) \, S(Y,K) \Big] . \end{aligned} \tag{54}$$

After normalization with the single particle spectra we thus recover the correct expression (1) for the correlator in the limit of vanishing final state interactions.

If the free propagation between emission points is neglected, according to (48c) all four terms have the same energies $\frac{1}{2}(k^0 + k^{0\prime}) = K^0$, $k^0 - k^{0\prime} = 0$, and it is obvious that it is not possible to recover an expression with any close similarity to (54). The phases resulting from the time evolution of the first particle until the appearance of the second one are thus crucial for the correct free-particle limit. One may be able to modify the propagation between emission points, but not to drop it completely. The correct free particle limit requires the appearance of a phase $\sim (t_a - t_b)$.

C. The smoothness approximation

For practical applications one would like to know how to combine this formalism with classical event generators which determine the emission functions $S(x, K)$ as

a set of last interaction points in the kinetic evolution the many-particle collision system. For particles without FSI it was shown in [3] that the symmetrized two particle cross section (54) can be computed by sampling the generated particles at the on-shell momenta $(\mathbf{p}_a, E_a)$, $(\mathbf{p}_b, E_b)$ for the direct term, and at the on-shell value $(\mathbf{K}, E_K)$ for the exchange term (with both selected particles having the *same* on-shell momentum $(\mathbf{K}, E_K)$!). The pairs selected for the exchange term are then multiplied with a "Bose-Einstein weight factor" $\cos q\cdot(x_i - x_j)$ where q is the relative momentum value at which we wish to know the correlator (*not* that of the selected pairs which have $p_i = p_j = K$), and $x_i - x_j$ is the space-time distance between the particles in the selected pairs. This does no longer work in the presence of FSI. Equations (48a), (50) show that now we need to sample the emission function $S(x,p)$ at all possible and in general off-shell values of the momentum p. The particles become only on-shell when they reach the detector, by virtue of the FSI. It is thus basically impossible to simulate Eqs. (48a), (50) with classical event generators unless one imposes a further approximation which essentially eliminates all off-shell effects.

This step is known as the smoothness approximation in which one assumes that $S(x, K \pm p)$ has a sufficiently weak momentum dependence that, over the p-range where $W_{\mathbf{q}/2}(X-Y,p)$ in Eq. (50) is non-vanishing, $S(x, K\pm p)$ can be replaced by either $S(x,K)$ or $S(x, K\pm p)$, depending on what seems more convenient. The domain of support of the function $W_{\mathbf{q}/2}(X-Y,p)$ is a measure of the ability of the FSI to change the particle momenta after freeze-out. For our purposes the smoothness approximation can be considered reliable as long as the typical shift in momentum in the FSI is small compared to the $\mathbf{q}$-range over which the correlator shows interesting structure. Note that for massive particles the FSI cause mostly a change of the spatial momentum while the corresponding energy transfer is very small. This is the basic reason why off-shell effects are small and why the smoothness approximation works.

Our implementation of the smoothness approximation differs in a crucial detail from previous approaches [7,8,11,12]. By formulating the approximation concisely and implementing it systematically we obtain an expression which shows a strong similarity to the free particle case in that it evaluates for the direct and exchange terms the source function at different momenta. In this way we automatically avoid some of the possible pathologies in the behaviour of the correlator [3,20,21] which may arise from the more traditional versions of the smoothness approximation.

We start from Eq. (48) which we rewrite as

$$
\begin{aligned}
P_2(\mathbf{p}_a,\mathbf{p}_b) &= 2\int d^4x\, d^4y\, \theta(y^0) \\
&\times \int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\, \Phi\left(\tfrac{\mathbf{q}}{2},\mathbf{k}'\right)\, \Phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) \\
&\times e^{-i[\omega(\mathbf{K}-\mathbf{k})-\omega(\mathbf{K}-\mathbf{k}')]y^0}\, e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{y}} \\
&\times S\left(x+\tfrac{y}{2}, K+\tfrac{k+k'}{2}\right)\, S\left(x-\tfrac{y}{2}, K-\tfrac{k+k'}{2}\right) . \qquad (55)
\end{aligned}
$$

Let us expand the frequency difference in the temporal phase factor for small values of $\mathbf{k},\mathbf{k}'$, keeping terms up to second order:

$$
\begin{aligned}
&[\omega(\mathbf{K}-\mathbf{k}) - \omega(\mathbf{K}-\mathbf{k}')] \\
&\approx -\frac{1}{E_K}(\mathbf{k}-\mathbf{k}')\cdot\left(\mathbf{K}-\frac{\mathbf{k}+\mathbf{k}'}{2}\right) . \qquad (56)
\end{aligned}
$$

Using (31), the two-particle spectrum thus becomes

$$
\begin{aligned}
P_2(\mathbf{p}_a,\mathbf{p}_b) &= \int d^4x\, d^4y\, \theta(y^0) \\
&\times \int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\, e^{-i(\mathbf{k}-\mathbf{k}')\cdot\left(\mathbf{y}-\frac{1}{E_K}\left(\mathbf{K}-\frac{\mathbf{k}+\mathbf{k}'}{2}\right)y^0\right)} \\
&\times\Big[\tilde\phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right)\tilde\phi\left(\tfrac{\mathbf{q}}{2},\mathbf{k}'\right) \pm \tilde\phi^*\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right)\tilde\phi\left(-\tfrac{\mathbf{q}}{2},\mathbf{k}'\right) \\
&\pm\tilde\phi^*\left(-\tfrac{\mathbf{q}}{2},\mathbf{k}\right)\tilde\phi\left(\tfrac{\mathbf{q}}{2},\mathbf{k}'\right) + \tilde\phi^*\left(-\tfrac{\mathbf{q}}{2},\mathbf{k}\right)\tilde\phi\left(-\tfrac{\mathbf{q}}{2},\mathbf{k}'\right)\Big] \\
&\times S\left(x+\tfrac{y}{2}, K+\tfrac{k+k'}{2}\right)\, S\left(x-\tfrac{y}{2}, K-\tfrac{k+k'}{2}\right) . \qquad (57)
\end{aligned}
$$

The functions $\tilde\phi$ describe the probability amplitude of finding in the FSI distorted wave with asymptotic relative momentum $\pm\mathbf{q}/2$ a plane wave with momentum $\mathbf{k}$ resp. $\mathbf{k}'$. These functions are peaked around $\mathbf{k},\mathbf{k}' = \pm\mathbf{q}/2$, and the peaking is the stronger the weaker the final state interactions are. The four terms in the square bracket in (57) thus peak at $(k{+}k')/2 = q/2, 0, 0$, and $-q/2$, respectively. (One easily checks that these equations hold for the corresponding 4-vectors.) We will use this property by replacing $(k+k')/2$ in the phase factor and in the arguments of the emission functions by the corresponding peak locations. This allows to pull the emission functions outside the $\mathbf{k},\mathbf{k}'$ integrations; it also removes the quadratic dependence of the phase factor on $\mathbf{k},\mathbf{k}'$ in favor of a linear one, turning the integrations over $\mathbf{k}$ and $\mathbf{k}'$ into normal Fourier integrals. The Fourier integrals can be performed, giving the corresponding relative wave functions in coordinate space:

$$
\begin{aligned}
&P_2(\mathbf{p}_a,\mathbf{p}_b) = \int d^4y\, \theta(y^0) \qquad (58) \\
&\Big[\left|\phi_{\mathbf{q}/2}(\mathbf{y}{-}\mathbf{v}_b y^0)\right|^2 \int d^4x\, S\left(x+\tfrac{y}{2}, p_a\right)\, S\left(x-\tfrac{y}{2}, p_b\right) \\
&\pm 2\,\mathrm{Re}\left(\phi^*_{\mathbf{q}/2}(\mathbf{y}{-}\mathbf{v}y^0)\phi_{-\mathbf{q}/2}(\mathbf{y}{-}\mathbf{v}y^0)\right) \\
&\qquad\qquad \times \int d^4x\, S\left(x+\tfrac{y}{2}, K\right)\, S\left(x-\tfrac{y}{2}, K\right) \\
&+ \left|\phi_{-\mathbf{q}/2}(\mathbf{y}{-}\mathbf{v}_a y^0)\right|^2 \int d^4x\, S\left(x+\tfrac{y}{2}, p_b\right)\, S\left(x-\tfrac{y}{2}, p_a\right)\Big] .
\end{aligned}
$$

Here we defined the three velocities

$$
\mathbf{v} = \frac{\mathbf{K}}{E_K}\,, \quad \mathbf{v}_a = \frac{\mathbf{p}_a}{E_K}\,, \quad \mathbf{v}_b = \frac{\mathbf{p}_b}{E_K} \qquad (59)
$$

associated with the observed particle momenta $\mathbf{p}_a$, $\mathbf{p}_b$, and their average $\mathbf{K}$.

Using the relations (27) one shows that in the middle term in (58) the factor containing the FSI relative wave functions is even under a simultaneous sign change of $\mathbf{y}$ and y^0. The product of emission functions in this term is even under the same sign change, too, which means that in this term we can replace $\theta(y^0)$ by $\frac{1}{2}[\theta(y^0)+\theta(-y^0)] = \frac{1}{2}$. The first and last term in (58) do not have such a symmetry, but can be combined by substituting $y \to -y$ in the last term to yield the final expression

$$\begin{aligned} P_2(\mathbf{p}_a, \mathbf{p}_b) &= \int d^4x\, d^4y\, S\left(x+\tfrac{y}{2}, p_a\right)\, S\left(x-\tfrac{y}{2}, p_b\right) \\ &\times \left[\theta(y^0)\left|\phi_{\mathbf{q}/2}(\mathbf{y}-\mathbf{v}_b y^0)\right|^2 + \theta(-y^0)\left|\phi_{\mathbf{q}/2}(\mathbf{y}-\mathbf{v}_a y^0)\right|^2\right] \\ &\pm \int d^4x\, d^4y\, S\left(x+\tfrac{y}{2}, K\right)\, S\left(x-\tfrac{y}{2}, K\right) \\ &\qquad \times \phi^*_{-\mathbf{q}/2}(\mathbf{y}-\mathbf{v}y^0)\, \phi_{\mathbf{q}/2}(\mathbf{y}-\mathbf{v}y^0). \end{aligned} \tag{60}$$

Once again one checks that in the limit of vanishing FSI (i.e. by replacing the functions ϕ by plane waves), and using the approximation $K^0 \approx E_K$ in accordance with the approximation made in Eq. (56), the correct expression (54) is recovered.

It is worthwhile to discuss the physical meaning of the arguments at which the FSI distorted waves ϕ in (60) must be evaluated. The sketch presented in Fig. 1 shows the relevant possibilities for the first term in (60) (the "direct term"). In this term the two emission functions are evaluated at the observed momenta p_a, p_b, corresponding to particle velocities $\mathbf{v}_a, \mathbf{v}_b$. Fig. 1 shows that in each case (i.e. for both possible time orderings between the two emission points) the argument of the FSI distorted wave ϕ corresponds to the spatial distance of the two particles *at the time when the second particle freezes out*, i.e. when the *pair* first exists. The different velocities which arise for the two different time orderings reflect the velocity of the earlier emitted particle in each case. In the second term of (60) (the "exchange term") the two emission functions are evaluated at the *same* momentum, namely the average pair momentum K, and correspondingly for both time orderings the earlier emitted particle has the velocity $\mathbf{v}$ corresponding to this momentum. Again the argument of the FSI distorted wave is the spatial distance between the two particles at the time of emission of the second particle.

D. Implementation in event generators

Equation (60) can be easily implemented into event generators, following essentially the same procedure as given in Ref. [3]:

For the *direct term* one selects all pairs (i, j) with $p_i = p_a$, $p_j = p_b$ within a given numerical accuracy (bin width) which is essentially dictated by event statistics. Each pair is multiplied with a weight given by the corresponding probability density $|\phi_{\mathbf{q}/2}|^2$ of the FSI distorted wave. The latter must be evaluated in a frame in which the pair moves non-relativistically, best in the pair rest frame where $\mathbf{K} = (\mathbf{p}_a + \mathbf{p}_b)/2 = 0$. (Then $E_K = m$, and the velocities (59) reduce to their usual nonrelativistic definition.) From the space-time coordinates x_i, x_j of the pair in the event generator frame one calculates the distance $\mathbf{y}^*_{ij}$ between the two particles in the pair rest frame at the time when the second particle is produced. One then computes $|\phi_{\mathbf{q}^*/2}(\mathbf{y}^*_{ij})|^2$ and weights the selected pair (i, j) with this number. In this expression $\mathbf{q}^*$ is the spatial relative momentum between the two particles in the pair rest frame which must be computed from p_a, p_b in the event generator frame. (We remark that in the absence of FSI, the corresponding weight is simply 1.) The complete direct term is obtained by summing over all such pairs.

For the *exchange term*, the selection of pairs and weights is a little less intuitive [3]: One selects all pairs (i, j) with $p_i = p_j = K$ (i.e. *on-shell particles* (!) with $\mathbf{p}_i = \mathbf{p}_j = \mathbf{K}$ and $E_i = E_j = E_K$), again within the same numerical accuracy (bin width) as above. From the production coordinates x_i, x_j one again computes the spatial distance $\mathbf{y}^*_{ij}$ between the two particles in the pair at the time of emission of the second one, in the pair rest frame $\mathbf{K} = 0$. This distance is used to compute the weight $\phi^*_{-\mathbf{q}^*/2}(\mathbf{y}^*_{ij})\phi^*_{\mathbf{q}^*/2}(\mathbf{y}^*_{ij})$ for this pair. The value of $\mathbf{q}^*$ here is *the same as above in the direct term*, i.e. it is computed from p_a and p_b by transforming into the pair rest frame, not from $p_i = p_j = K$. (Without FSI, the corresponding weight would be $\cos(\mathbf{q}^* \cdot \mathbf{y}^*_{ij})$ [3].) The full exchange term is obtained by summing over all such pairs.

Finally one must normalize the correlator by the product of single particle spectra,

$$P_1(\mathbf{p}_a)\, P_1(\mathbf{p}_b) = \int d^4x\, S(x, p_a) \int d^4y\, S(y, p_b)\,. \tag{61}$$

This normalization is best obtained from the pairs selected for the direct term above by multiplying them with unit weights.

One may object to the use of event generators for the emission function because they fix particle momenta and coordinates simultaneously and thus violate the uncertainty principle. It was shown in [3] how to generate from an event generator a quantum mechanically consistent Wigner density $S(x, p)$ by folding the event generator output with minimum uncertainty wave packets. The corresponding quantum mechanically consistent algorithm for computing single- and two-particle spectra given in [3] is easily generalized to include FSI effects, by simply replacing the factors 1 and $\cos(\mathbf{q}^* \cdot \mathbf{y}^*_{ij})$ in the direct and exchange terms, respectively, by the correct FSI weights as discussed above.

E. Comparison to previously published expressions

In previous treatments [7,8,11,21] the smoothness approximation was implemented in a different way: the smoothness of $S(x,p)$ as a function of momentum was used to replace the momentum arguments of the two emission functions in both the direct and the exchange term of Eqs. (54), (60) in exactly the same way, namely either by p_a in the first and by p_b in the second, or by K in both emission functions. The first of these two alternatives can, for sources with very strong x-p-correlations, lead to correlators with pathological behaviour, as discussed in [3,20,21]. The second alternative was exploited in the FSI studies presented in Refs. [7,8,11]. In this case both the direct and the exchange term in (60) involve the same combination of emission functions, namely

$$D(y,K) \equiv \int d^4x\, S\left(x+\tfrac{y}{2},K\right)\, S\left(x-\tfrac{y}{2},K\right) . \qquad (62)$$

$D(y,K)$ is the "relative distance distribution" of the source, i.e. the distribution of relative space-time distances y between the particles in emitted pairs with momentum K. Obviously D is an even function of y, $D(y,K)=D(-y,K)$. Also, since in the direct term of (60) the emission function arguments p_a, p_b are now both replaced by K, the velocities $\mathbf{v}_a, \mathbf{v}_b$ in the corresponding FSI distorted wave factors must also both be replaced by $\mathbf{v}$. The two time orderings can then be combined by using $\theta(y^0)+\theta(-y^0)=1$, and we obtain

$$P_2(\mathbf{p}_a,\mathbf{p}_b) = P_2(\mathbf{q},\mathbf{K}) \approx \int d^4y\, w(y;q,K)\, D(y,K)\,, \qquad (63)$$

with the "weight function"

$$\begin{aligned} w(y;q,K) &= \left|\Phi_{\mathbf{q}/2}(\mathbf{y}-\mathbf{v}y^0)\right|^2 \qquad (64)\\ &= \frac{1}{2}\left|\phi_{\mathbf{q}/2}(\mathbf{y}-\mathbf{v}y^0) \pm \phi_{\mathbf{q}/2}(-(\mathbf{y}-\mathbf{v}y^0))\right|^2 . \end{aligned}$$

[Note that if P_2 is approximated as in (63) one should also for consistency use the corresponding approximation $P_1(\mathbf{p}_a)P_1(\mathbf{p}_b) = \left[P_1(\mathbf{K})\right]^2$ for the normalization of the correlator.] Eqs. (63), (64) are identical with Eq. (2.11) in Ref. [11]. In the pair rest system ($\mathbf{v}=0=\mathbf{K}$) P_2 reduces to

$$P_2(\mathbf{p}_a,\mathbf{p}_b) \approx \int d^3y\, \left|\Phi_{\mathbf{q}/2}(\mathbf{y})\right|^2 \int dy^0\, D(y,K)\,. \qquad (65)$$

This expression was first written down by Koonin [22] and has recently been used in [23]. Note that on the r.h.s. of (65) everything must be evaluated in the pair rest system with $\mathbf{K}=0$. In this version of the smoothness approximation the two-particle spectrum $P_2(\mathbf{q},\mathbf{K})$ is thus given by the time integrated relative distance distribution in the pair rest system defined by $\mathbf{K}$, weighted with the probability density of the outgoing distorted wave with relative momentum $\mathbf{q}$ at the spatial relative distance at which the pair was created.

F. Nonidentical particles

For correlations introduced by FSI between non-identical pairs one has to take into account two differences:

(1) the two masses m_a and m_b are unequal, and

(2) the two-particle wave-functions are not symmetrized resp. antisymmetrized.

The first modification requires the introduction of correspondingly modified center-of-mass and relative coordinates in several steps of the derivation given in Sec. II. Still, the problem remains non-relativistic in any system in which the pair is at rest or moves non-relativistically, and the rest of the derivation goes through as before. The final result is again given by Eq. (60); the only difference is that now $\Phi_{\mathbf{q}/2}(\mathbf{y})$ is just the distorted wave for the relative motion including FSI, not its symmetrized form (31). As a result of the mass asymmetry, the weight function $w(y;q,K)$ even in the pair rest system will no longer be symmetric in $\mathbf{y}$. Its asymmetry will be reflected in an asymmetry of the correlator under sign change of $\mathbf{q}$ which can be used to extract information about the average distance between the effective sources from which the two particle species originate [24].

G. Coulomb final state interactions

For completeness and to correct a few confusing typographical errors in Ref. [7] we discuss explicitly the distorted wave for Coulomb final state interacions. In this case the solution of the Schrödinger Eq. (17) is

$$\phi_{\pm\mathbf{q}/2}(\mathbf{r}) = \Gamma(1+i\eta)e^{-\frac{1}{2}\pi\eta}\, e^{\pm\frac{i}{2}\mathbf{q}\cdot\mathbf{r}} F(-i\eta;1;iz_\mp)\,, \qquad (66)$$

where ($q=|\mathbf{q}|$, $r=|\mathbf{r}|$)

$$\eta = \frac{\alpha\mu}{q/2} = \frac{\alpha m}{q} \qquad (67)$$

is the Sommerfeld parameter, and

$$z_\mp = \tfrac{1}{2}\left(qr \mp \mathbf{q}\cdot\mathbf{r}\right) = \tfrac{1}{2}qr(1\mp\cos\theta)\,, \qquad (68)$$

θ being the angle between $\mathbf{q}$ and $\mathbf{r}$. $F(-i\eta;1;iz)$ is a confluent hypergeometric function with the series expansion

$$\begin{aligned} F(-i\eta;1;iz) &= \sum_{n=0}^{\infty} \frac{\Gamma(n-i\eta)}{\Gamma(-i\eta)}\,\frac{(iz)^n}{(n!)^2} \\ &= 1 - i\eta\sum_{n=1}^{\infty}(1-i\eta)(2-i\eta)\cdots(n-1-i\eta)\frac{(iz)^n}{(n!)^2} . \qquad (69) \end{aligned}$$

For small $\eta \ll 1$ this can be approximated by [7]

$$\begin{aligned} F(-i\eta;1;iz) &\approx 1 - i\eta\sum_{n=1}^{\infty}\frac{(iz)^n}{n\cdot n!} \\ &= 1 - i\eta\Big(\mathrm{Ci}(z) + i\,\mathrm{Si}(z) - \ln z - \gamma\Big) . \qquad (70) \end{aligned}$$

This is good for pions with $q \gg 1$ MeV and for protons with $q \gg 7$ MeV. The square of the normalization factor in (66) yields the well-known Gamov factor

$$G(q) \equiv \frac{2\pi\eta}{e^{2\pi\eta}-1} = e^{-\pi\eta}\,|\Gamma(1+i\eta)|^2\,. \tag{71}$$

With these expressions the two-particle cross section (60) takes the form

$$P_2(\mathbf{q},\mathbf{K}) = G(q)\left[I_1(\mathbf{q},\mathbf{K}) \pm I_2(\mathbf{q},\mathbf{K})\right], \tag{72a}$$

where the direct term is given by

$$\begin{aligned} I_1(\mathbf{q},\mathbf{K}) = \int d^4y \Big[\theta(y^0)\, \left|F\left(-i\eta;1;iz_-^b(y^0)\right)\right|^2 \\ +\theta(-y^0)\, \left|F\left(-i\eta;1;iz_-^a(y^0)\right)\right|^2\Big] \\ \times \int d^4x\, S\left(x+\tfrac{y}{2},K+\tfrac{q}{2}\right)\, S\left(x-\tfrac{y}{2},K-\tfrac{q}{2}\right) \end{aligned} \tag{72b}$$

while the exchange term is

$$\begin{aligned} I_2(\mathbf{q},\mathbf{K}) = \int d^4y\, e^{i\mathbf{q}\cdot(\mathbf{y}-\mathbf{v}y^0)}\, D(y,K) \\ \times F\left(i\eta;1;-iz_+(y^0)\right)\, F\left(-i\eta;1;iz_-(y^0)\right)\,. \end{aligned} \tag{72c}$$

Here $D(y,K)$ is the relative distance distribution (62), and $z_-^b(y^0)$, $z_-^a(y^0)$, $z_\pm(y^0)$ are determined with the help of (68) by substituting $\mathbf{r} \to \mathbf{y}-\mathbf{v}_b y^0$, $\mathbf{y}-\mathbf{v}_a y^0$, and $\mathbf{y}-\mathbf{v}y^0$, respectively. Note that the integrals $I_{1,2}$ should be evaluated in (or close to) the frame where $\mathbf{K}=\mathbf{v}=0$.

Equations (72) correct Eq. (3.11) in Ref. [7].

IV. CONCLUSIONS

Let us shortly summarize our main results:

We studied the effect of two-body final state interactions on the two-particle coincidence cross section, both for pairs of identical and of non-identical particles. We used only two approximations: (1) the source is chaotic, and (2) the total particle multiplicity is large. The first of these two assumptions is crucial since it allows to factorize the two-particle Wigner density of the source and express the two-particle cross section in terms of the single-particle Wigner density, i.e. the emission function $S(x,p)$ of the source.

From these two assumptions we derived, without further approximation, the general expression (48) for the two-particle cross section in terms of the emission function $S(x,p)$ and the FSI distorted relative wave functions in momentum space, $\Phi\left(\frac{\mathbf{q}}{2},\mathbf{k}\right)$. This expression can be rewritten in the generic Wigner representation (50). In spite of its generality, this expression is of limited practical usefulness because it involves the emission function at arbitrary of-shell momenta which is usually not known. Since for massive particles the FSI shifts, however, mostly the spatial momenta of the particles while the corresponding energy transfer is very small, it is in practice possible to replace the off-shell momenta in the emission function by on-shell values. We do this systematically in the framework of the so-called smoothness approximation which we discussed in Sec. III C. Here our treatment differs from previously published ones, and our formula (60) thus improves upon known results.

Eq. (60) involves only on-shell momenta and can thus be implemented in classical event generators. This was discussed in Sec. III D. In contrast to previously published expressions, our result exhibits the same asymmetry in the momentum arguments of the emission functions between the direct and exchange terms as expression (54) for the free case to which it correctly reduces when the FSI are switched off.

This asymmetry between the direct and exchange terms is eliminated if one implements the more stringent version of the smoothness approximation used by Pratt and collaborators which replaces the momentum argument of the emission function by the average pair momentum K everywhere. In Sec. III E we showed how in this way the well-known Koonin-Pratt formula, Eqs. (63)-(65), is recovered.

The differences between the "correct" asymmetric form and the "approximate" symmetric Koonin-Pratt form were recently extensively investigated for the case of free particle propagation without FSI after freeze-out. They were found to be potentially severe for sources with strong x-p-correlations (e.g. sources which feature strong collective expansion) [3,20], but the problem appears to be less serious if the source is sufficiently large [21]. While the existence of the "correct" asymmetric expression (60) (for Coulomb FSI an explicit expression was given in (72)) now eliminates the need for using the approximate and somewhat uncertain Koonin-Pratt formula, it would still be nice to have a quantitative feeling for the error margin associated with the Koonin-Pratt approximation. A systematic numerical study of this question which includes FSI effects is under way.

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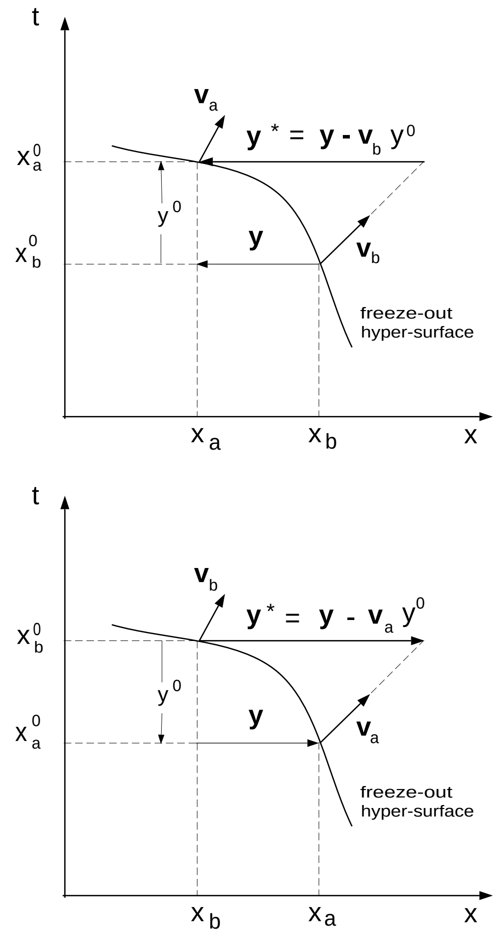

FIG. 1. Graphical illustration of the coordinates at which the FSI distorted waves ϕ in (60) are evaluated. Upper panel: the case $y^0 > 0$ (i.e. $x_a^0 > x_b^0$). Lower panel: the case $y^0 < 0$ (i.e. $x_a^0 < x_b^0$).